Glossary

A

Absolute maximum and minimum [p. 455]
For a continuous function f defined on an interval $[a, b]$:

- the *absolute maximum* is the value M of the function f such that $f(x) \le M$ for all $x \in [a, b]$
- the *absolute minimum* is the value N of the function f such that $f(x) \ge N$ for all $x \in [a, b]$.

Acceleration [p. 319] the rate of change of a particle's velocity with respect to time

Acceleration, average [p. 319] The average acceleration of a particle for the time interval $[t_1, t_2]$ is given by $\dfrac{v_2 - v_1}{t_2 - t_1}$, where v_2 is the velocity at time t_2 and v_1 is the velocity at time t_1.

Acceleration, instantaneous [pp. 319, 442]

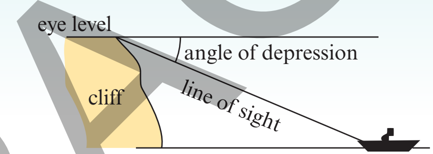
$$a = \frac{dv}{dt} = \frac{d^2x}{dt^2}$$

Addition rule for choices [p. 743] To determine the total number of choices from disjoint alternatives, simply add up the number of choices available for each alternative.

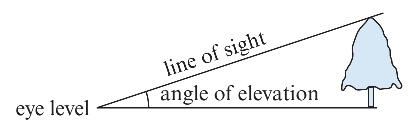
Addition rule for probability [p. 518] The probability of A or B or both occurring is given by
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

Amplitude of trigonometric functions [p. 140]
The distance between the mean position and the maximum position is called the amplitude.
The graph of $y = \sin x$ has an amplitude of 1.

Angle of depression [p. 497] the angle between the horizontal and a direction below the horizontal



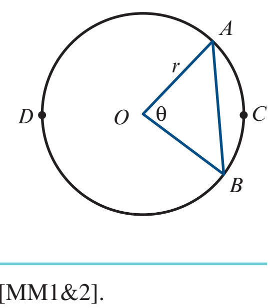
Angle of elevation [p. 497] the angle between the horizontal and a direction above the horizontal



Anti-derivative [p. 347] To find the general anti-derivative of $f(x)$: If $F'(x) = f(x)$, then
$\int f(x)\,dx = F(x) + c$
where c is an arbitrary real number.

Arc [MM1&2] Two points on a circle divide the circle into arcs; the shorter is the *minor arc*, and the longer is the *major arc*.

Arc length, ℓ [MM1&2]
The length of arc ACB is given by $\ell = r\theta$, where $\theta^c = \angle AOB$.

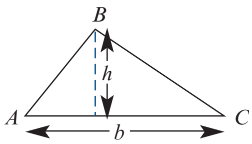


Note: The glossary contains some terms from Mathematical Methods Units 1 & 2 [MM1&2].

Area of a triangle [p. 494] given by half the product of the lengths of two sides and the sine of the angle included between them.

$$\text{Area} = \frac{1}{2}bh$$

$$\text{Area} = \frac{1}{2}bc\sin A$$



Arithmetic sequence [MM1&2] a sequence in which each successive term is found by adding a fixed amount to the previous term; e.g. $2, 5, 8, 11, \ldots$. An arithmetic sequence has a recurrence relation of the form $t_n = t_{n-1} + d$, where d is the common difference. The nth term can be found using $t_n = a + (n-1)d$, where $a = t_1$.

Arithmetic series [MM1&2] the sum of the terms in an arithmetic sequence. The sum of the first n terms is given by the formula

$$S_n = \frac{n}{2}\big(2a + (n-1)d\big)$$

where $a = t_1$ and d is the common difference.

Arrangements [p. 744] counted when order is important. The number of ways of selecting and arranging r objects from a total of n objects is

$$\frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \cdots \times (n-r+1)$$

Asymptote [MM1&2] A straight line is an asymptote of the graph of a function $y = f(x)$ if the graph of $y = f(x)$ gets arbitrarily close to the straight line. An asymptote can be horizontal, vertical or oblique.

Average value [p. 402] The average value of a continuous function f for an interval $[a, b]$ is defined as $\frac{1}{b-a}\int_a^b f(x)\,dx$.

B

Bearing [p. 498] the compass bearing; the direction measured from north clockwise

Bernoulli random variable [p. 560] a random variable that takes only the values 1 (indicating a success) and 0 (indicating a failure)

Bernoulli sequence [p. 560] a sequence of repeated trials with the following properties:

- Each trial results in one of two outcomes, usually designated as a success or a failure.
- The probability of success on a single trial, p, is constant for all trials.
- The trials are independent. (The outcome of a trial is not affected by outcomes of other trials.)

Binomial distribution [p. 562] The probability of observing x successes in n independent trials, each with probability of success p, is given by

$$\Pr(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

where $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

Binomial expansion [p. 748]

$$(x+a)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} a^k$$

$$= x^n + \binom{n}{1} x^{n-1} a + \binom{n}{2} x^{n-2} a^2 + \cdots + a^n$$

The $(r+1)$st term is $\binom{n}{r} x^{n-r} a^r$.

Binomial experiment [p. 562] a Bernoulli sequence of n independent trials, each with probability of success p

C

Chain rule [p. 282] The chain rule can be used to differentiate a complicated function $y = f(x)$ by transforming it into two simpler functions, which are 'chained' together:

$$x \xrightarrow{h} u \xrightarrow{g} y$$

Using Leibniz notation, the chain rule is stated as

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Change of base [p. 211] $\log_a b = \frac{\log_c b}{\log_c a}$

Circle, general equation [p. 7] The general equation for a circle is $(x-h)^2 + (y-k)^2 = r^2$, where the centre is (h, k) and the radius is r.

Coefficient [p. 90] the number that multiplies a power of x in a polynomial. E.g. for $2x^5 - 7x^2 + 4$, the coefficient of x^2 is -7.

Combinations [p. 744] *see* selections

Common difference, *d* [MM1&2] the difference between two consecutive terms of an arithmetic sequence, i.e. $d = t_n - t_{n-1}$

Common ratio, *r* [MM1&2] the quotient of two consecutive terms of a geometric sequence, i.e.

$$r = \frac{t_n}{t_{n-1}}$$

Compass bearing [p. 498] the direction measured from north clockwise

Complement, *A′* [p. 518] the set of outcomes that are in the sample space, ε, but not in A. The probability of the event A' is $\Pr(A') = 1 - \Pr(A)$.

Complementary relationships [p. 138]

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta \qquad \sin\left(\frac{\pi}{2} + \theta\right) = \cos\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta \qquad \cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$$

Composition of functions [p. 23] For two functions f and g, the function with rule $h(x) = f(g(x))$ is the composition of f with g. We write $h = f \circ g$. For example, if $f(x) = x^4$ and $g(x) = x + 1$, then $(f \circ g)(x) = f(g(x)) = (x + 1)^4$.

Compound interest [MM1&2] is calculated at regular intervals on the total of the amount originally invested and the interest accumulated over the previous years. If \$$P$ is invested at R% p.a. compounded annually, then the value of the investment after n years, \$$A_n$, is given by

$$A_n = Pr^n, \text{ where } r = 1 + \frac{R}{100}$$

Concavity [p. 447]
- If $f''(x) > 0$ for all $x \in (a, b)$, then the gradient of the curve is increasing over the interval; the curve is said to be *concave up*.
- If $f''(x) < 0$ for all $x \in (a, b)$, then the gradient of the curve is decreasing over the interval; the curve is said to be *concave down*.

Conditional probability [p. 526] the probability of an event A occurring when it is known that some event B has occurred, given by

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Confidence interval [p. 678] an interval estimate for the population proportion p based on the value of the sample proportion $\hat{p}$

Congruence tests [p. 480] Two triangles are congruent if one of the following conditions holds:
- **SSS** the three sides of one triangle are equal to the three sides of the other triangle
- **SAS** two sides and the included angle of one triangle are equal to two sides and the included angle of the other triangle
- **AAS** two angles and one side of one triangle are equal to two angles and the matching side of the other triangle
- **RHS** the hypotenuse and one side of a right-angled triangle are equal to the hypotenuse and one side of another right-angled triangle.

Congruent figures [p. 480] have exactly the same shape and size

Constant function [MM1&2] a function with a rule of the form $f(x) = a$; e.g. $f(x) = 7$

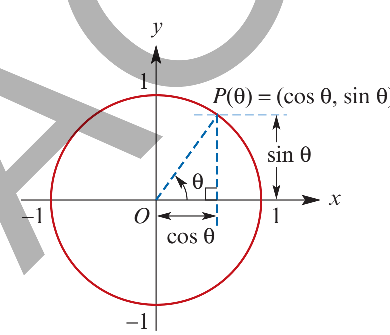
Continuous function [p. 274] A function f is continuous at the point $x = a$ if $\lim_{x \to a} f(x) = f(a)$.

Continuous random variable [p. 582] a random variable X that can take any value in an interval of the real number line

Convergent series [MM1&2] An infinite series $t_1 + t_2 + t_3 + \cdots$ is convergent if the sum of the first n terms, S_n, approaches a limiting value as $n \to \infty$. An infinite geometric series is convergent if $-1 < r < 1$, where r is the common ratio.

Coordinates [MM1&2] an ordered pair of numbers that identifies a point in the Cartesian plane; the first number identifies the position with respect to the x-axis, and the second number identifies the position with respect to the y-axis

Cosine function [p. 131] cosine θ is defined as the x-coordinate of the point P on the unit circle where OP forms an angle of θ radians with the positive direction of the x-axis.

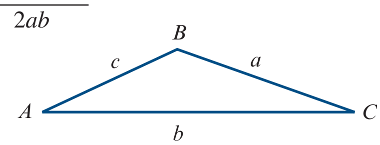


Cosine rule [p. 490] For triangle ABC:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or equivalently

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



The cosine rule is used to find unknown quantities in a triangle given two sides and the included angle, or given three sides.

Cubic function [p. 103] a polynomial of degree 3. A cubic function f has a rule of the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

Cumulative distribution function [p. 610] gives the probability that the random variable X takes a value less than or equal to x; that is,

$$F(x) = \Pr(X \leq x) = \int_{-\infty}^{x} f(t)\, dt$$

D

Definite integral [pp. 378, 384] $\int_a^b f(x)\,dx$ denotes the signed area enclosed by the graph of $y = f(x)$ between $x = a$ and $x = b$.

Degree of a polynomial [p. 90] given by the highest power of x with a non-zero coefficient; e.g. the polynomial $2x^5 - 7x^2 + 4$ has degree 5.

Dependent variable [MM1&2] If one variable, y, can be expressed as a function of another variable, x, then the value of y depends on the value of x. We say that y is the *dependent variable* and that x is the *independent variable*.

Derivative function [p. 256] also called the gradient function. The derivative f' of a function f is given by

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Derivatives, basic [pp. 259, 271, 285–296]

$f(x)$	$f'(x)$	
c	0	where c is a constant
x^a	ax^{a-1}	where $a \in \mathbb{R} \setminus \{0\}$
e^{kx}	ke^{kx}	
$\ln(kx)$	$\frac{1}{x}$	
$\sin(kx)$	$k\cos(kx)$	
$\cos(kx)$	$-k\sin(kx)$	

Difference of two cubes [p. 99]
$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Difference of two squares [MM1&2]
$x^2 - y^2 = (x - y)(x + y)$

Differentiable [p. 275] A function f is said to be differentiable at the point $x = a$ if $\displaystyle\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$ exists.

Differentiation rules [p. 259]

- Sum: $f(x) = g(x) + h(x)$, $f'(x) = g'(x) + h'(x)$
- Multiple: $f(x) = k\,g(x)$, $f'(x) = k\,g'(x)$

see also chain rule, product rule, quotient rule

Dilation from the *x*-axis [p. 50] A dilation of factor b from the x-axis is described by the rule $(x, y) \to (x, by)$. The curve with equation $y = f(x)$ is mapped to the curve with equation $y = bf(x)$.

Dilation from the *y*-axis [p. 50] A dilation of factor a from the y-axis is described by the rule $(x, y) \to (ax, y)$. The curve with equation $y = f(x)$ is mapped to the curve with equation $y = f\left(\frac{x}{a}\right)$.

Discontinuity [p. 274] A function is said to be discontinuous at a point if it is not continuous at that point.

Discrete random variable [p. 534] a random variable X which can take only a countable number of values, usually whole numbers

Discriminant, Δ, of a quadratic [p. 82] the expression $b^2 - 4ac$, which is part of the quadratic formula. For the quadratic equation $ax^2 + bx + c = 0$:

- If $b^2 - 4ac > 0$, there are two solutions.
- If $b^2 - 4ac = 0$, there is one solution.
- If $b^2 - 4ac < 0$, there are no real solutions.

Disjoint [p. 2] Two sets A and B are disjoint if they have no elements in common, i.e. $A \cap B = \emptyset$.

Displacement [p. 316] The displacement of a particle moving in a straight line is defined as the change in position of the particle.

Distance between two points [p. 43] The distance between points $A(x_1, y_1)$ and $B(x_2, y_2)$ is $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Domain [p. 6] the set of all the first coordinates of the ordered pairs in a relation

E

Element [p. 2] a member of a set.

- If x is an element of a set A, we write $x \in A$.
- If x is *not* an element of a set A, we write $x \notin A$.

Empty set, ∅ [p. 2] the set that has no elements

Equating coefficients [p. 92] Two polynomials P and Q are equal only if their corresponding coefficients are equal. For example, two cubic polynomials $P(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ and $Q(x) = b_3x^3 + b_2x^2 + b_1x + b_0$ are equal if and only if $a_3 = b_3$, $a_2 = b_2$, $a_1 = b_1$ and $a_0 = b_0$.

Euler's number, *e* [p. 196] the natural base for exponential and logarithmic functions:

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = 2.718\,281\ldots$$

Even function [p. 18] A function f is even if $f(-x) = f(x)$ for all x in the domain of f; the graph is symmetric about the y-axis.

Event [p. 516] a subset of the sample space (that is, a set of outcomes)

Expected value of a random variable, E(*X*) [pp. 542, 595] also called the mean, μ.
For a discrete random variable X:

$$E(X) = \sum_x x \cdot \Pr(X = x) = \sum_x x \cdot p(x)$$

For a continuous random variable X:

$$E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

Exponential function [p. 190] a function $f(x) = ka^x$, where k is a non-zero constant and the base a is a positive real number other than 1

F

Factor [MM1&2] a number or expression that divides another number or expression without remainder

Factor theorem [p. 98] If $\beta x + \alpha$ is a factor of a polynomial $P(x)$, then $P\left(-\frac{\alpha}{\beta}\right) = 0$. Conversely, if $P\left(-\frac{\alpha}{\beta}\right) = 0$, then $\beta x + \alpha$ is a factor of $P(x)$.

Factorise [MM1&2] express as a product of factors

Formula [MM1&2] an equation containing symbols that states a relationship between two or more quantities; e.g. $A = \ell w$ (area = length × width). The value of A, the subject of the formula, can be found by substituting given values of ℓ and w.

Function [p. 8] a relation such that for each x-value there is only one corresponding y-value. This means that, if (a, b) and (a, c) are ordered pairs of a function, then $b = c$.

Function, vertical-line test [p. 8] used to identify whether a relation is a function or not. If a vertical line can be drawn anywhere on the graph and it only ever intersects the graph a maximum of once, then the relation is a *function*.

Fundamental theorem of calculus [p. 380] If f is a continuous function on an interval $[a, b]$, then
$\int_a^b f(x)\,dx = F(b) - F(a)$
where F is any anti-derivative of f and $\int_a^b f(x)\,dx$ is the definite integral from a to b.

G

Geometric sequence [MM1&2] a sequence in which each successive term is found by multiplying the previous term by a fixed amount; e.g. 2, 6, 18, 54, ... A geometric sequence has a recurrence relation of the form $t_n = rt_{n-1}$, where r is the common ratio. The nth term can be found using $t_n = ar^{n-1}$, where $a = t_1$.

Geometric series [MM1&2] the sum of the terms in a geometric sequence. The sum of the first n terms is given by the formula
$$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{a(1 - r^n)}{1 - r}$$
where $a = t_1$ and r is the common ratio.

Gradient function *see* derivative function

Gradient of a line [p. 43] The gradient is
$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$
where (x_1, y_1) and (x_2, y_2) are the coordinates of two points on the line. The gradient of a vertical line (parallel to the y-axis) is undefined.

I

Implied domain *see* maximal domain

Indefinite integral *see* anti-derivative

Independence [p. 529] Two events A and B are independent if and only if
$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$

Independent variable [MM1&2] If one variable, y, can be expressed as a function of another variable, x, then the value of y depends on the value of x. We say that y is the *dependent variable* and that x is the *independent variable*.

Index laws [p. 200]
- To multiply two powers with the same base, add the indices: $a^x \times a^y = a^{x+y}$
- To divide two powers with the same base, subtract the indices: $a^x \div a^y = a^{x-y}$
- To raise a power to another power, multiply the indices: $(a^x)^y = a^{x \times y}$
- Rational indices: $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$
- For base $a \in \mathbb{R}^+ \setminus \{1\}$, if $a^x = a^y$, then $x = y$.

Inequality [MM1&2] a mathematical statement that contains an inequality symbol rather than an equals sign; e.g. $2x + 1 < 4$

Infinite geometric series [MM1&2]
If $-1 < r < 1$, then the sum to infinity is given by
$$S_\infty = \frac{a}{1 - r}$$
where $a = t_1$ and r is the common ratio.

Integers [p. 3] $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Integrals, basic [pp. 348–358]

$f(x)$	$\int f(x)\,dx$	
x^r	$\frac{x^{r+1}}{r+1} + c$	where $r \in \mathbb{Q} \setminus \{-1\}$
$\frac{1}{ax+b}$	$\frac{1}{a}\ln(ax+b) + c$	for $ax + b > 0$
e^{kx}	$\frac{1}{k}e^{kx} + c$	
$\sin(kx)$	$-\frac{1}{k}\cos(kx) + c$	
$\cos(kx)$	$\frac{1}{k}\sin(kx) + c$	

Integration, properties [p. 348]
- $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$
- $\int kf(x)\,dx = k\int f(x)\,dx$

Integration (definite), properties [p. 388]
- $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$
- $\int_a^a f(x)\,dx = 0$
- $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

Intersection of sets [pp. 2, 516] The intersection of two sets A and B, written $A \cap B$, is the set of all elements common to A and B.

Interval [p. 4] a subset of the real numbers of the form $[a, b]$, $[a, b)$, (a, ∞), etc.

Irrational number [p. 3] a real number that is not rational; e.g. π and $\sqrt{2}$

Iterative rule [MM1&2] *see* recurrence relation

K

Karnaugh map [p. 521] a probability table

L

Law of total probability [p. 527] In the case of two events, A and B:
$\Pr(A) = \Pr(A \mid B)\Pr(B) + \Pr(A \mid B')\Pr(B')$

Left-endpoint method [p. 373] gives an estimate for the area under the graph of $y = f(x)$ between $x = a$ and $x = b$:

$$L_n = \frac{b-a}{n}\left[f(x_0) + f(x_1) + \cdots + f(x_{n-1})\right]$$

Limit [p. 255] The notation $\lim_{x \to a} f(x) = p$ says that the limit of $f(x)$, as x approaches a, is p. We can also say: 'As x approaches a, $f(x)$ approaches p.'

Linear equation [MM1&2] a polynomial equation of degree 1; e.g. $2x + 1 = 0$

Linear function [MM1&2] a function with a rule of the form $f(x) = mx + c$; e.g. $f(x) = 3x + 1$

Linear function of a random variable [p. 607]

- $\mathrm{E}(aX + b) = a\mathrm{E}(X) + b$
- $\mathrm{Var}(aX + b) = a^2\mathrm{Var}(X)$

Literal equation [p. 116] an equation for the variable x in which the coefficients of x, including the constants, are pronumerals; e.g. $ax + b = c$

Logarithm [p. 202] If $a \in \mathbb{R}^+ \setminus \{1\}$ and $x \in \mathbb{R}$, then the statements $a^x = y$ and $\log_a y = x$ are equivalent.

Logarithm, natural [p. 203] The natural logarithm function is given by
$\ln x = \log_e x$
where the base e is Euler's number.

Logarithm laws [p. 204]

- $\log_a(xy) = \log_a x + \log_a y$
- $\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$
- $\log_a\left(\frac{1}{x}\right) = -\log_a x$
- $\log_a(x^p) = p\log_a x$

Logarithmic scale [p. 227] a measurement scale that uses the logarithm of a quantity; e.g. Richter scale for earthquakes

M

Margin of error, E [p. 682] the distance between the sample estimate and the endpoints of the confidence interval

Maximal domain [p. 16] When the rule for a relation is given and no domain is specified, then the domain taken is the largest for which the rule has meaning.

Maximum and minimum value [p. 455] *see* absolute maximum and minimum

Mean of a random variable, μ [pp. 542, 595] *see* expected value of a random variable, $\mathrm{E}(X)$

Median of a random variable, m [p. 598] the middle value of the distribution. For a continuous random variable, the median is the value m such that $\int_{-\infty}^{m} f(x)\,dx = 0.5$.

Midpoint of a line segment [p. 43] If $P(x, y)$ is the midpoint of the line segment joining $A(x_1, y_1)$ and $B(x_2, y_2)$, then

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

Model [p. 236] a mathematical representation of a real-world situation. For example, an equation that describes the relationship between two physical quantities is a *mathematical model*.

Multiplication rule for choices [p. 743] When sequential choices are involved, the total number of possibilities is found by multiplying the number of options at each successive stage.

Multiplication rule for probability [p. 526] the probability of events A and B both occurring is
$\Pr(A \cap B) = \Pr(A \mid B) \times \Pr(B)$

Multi-stage experiment [p. 527] an experiment that could be considered to take place in more than one stage; e.g. tossing two coins

Mutually exclusive [p. 518] Two events are said to be mutually exclusive if they have no outcomes in common.

N

$n!$ [p. 744] read as 'n factorial', the product of all the natural numbers from n down to 1:
$n! = n \times (n-1) \times (n-2) \times (n-3) \times \cdots \times 2 \times 1$

Natural logarithm [p. 203] *see* logarithm

Natural numbers [p. 3] $\mathbb{N} = \{1, 2, 3, 4, \dots\}$

^nC_r [p. 744] the number of combinations of n objects in groups of size r:

$$^nC_r = \frac{n!}{r!\,(n-r)!}$$

An alternative notation for ^nC_r is $\binom{n}{r}$.

Normal distribution [p. 625] a symmetric, bell-shaped distribution that often occurs for a measure in a population (e.g. height, weight, IQ); its centre is determined by the mean, μ, and its width by the standard deviation, σ.

Normal line, equation [p. 305] Let (x_1, y_1) be a point on the curve $y = f(x)$. If f is differentiable at $x = x_1$, the equation of the normal at (x_1, y_1) is

$$y - y_1 = \frac{-1}{f'(x_1)}(x - x_1)$$

O

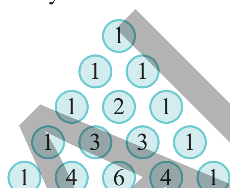
Odd function [p. 18] A function f is odd if $f(-x) = -f(x)$ for all x in the domain of f; the graph has rotational symmetry about the origin.

Optimisation problem [p. 458] a problem where a quantity is to be maximised or minimised under given constraints; e.g. to maximise the area of land enclosed by a fixed length of fencing

Ordered pair [p. 6] a pair of elements, denoted (x, y), where x is the first coordinate and y is the second coordinate

P

Pascal's triangle [p. 748] a triangular pattern of numbers formed by the binomial coefficients ^nC_r



Percentile [p. 597] For a continuous random variable X, the value p such that $\Pr(X \le p) = q\%$ is called the qth percentile of X, and is found by solving $\int_{-\infty}^{p} f(x)\,dx = \frac{q}{100}$.

Period of a function [p. 140] A function f with domain $\mathbb{R}$ is periodic if there is a positive constant a such that $f(x + a) = f(x)$ for all x. The smallest such a is called the period of f.

- Sine and cosine have period 2π.
- Tangent has period π.
- A function of the form $y = a\cos(nx + \varepsilon) + b$ or $y = a\sin(nx + \varepsilon) + b$ has period $\frac{2\pi}{n}$.

Permutations [p. 744] *see* arrangements

Piecewise-defined function [p. 17] a function which has different rules for different subsets of its domain

Point estimate [p. 678] If the value of the sample proportion $\hat{p}$ is used as an estimate of the population proportion p, then it is called a point estimate of p.

Point of inflection [p. 447] a point where a curve changes from concave up to concave down or from concave down to concave up. That is, a point of inflection occurs where the sign of the second derivative changes.

Polynomial function [p. 90] A polynomial has a rule of the type

$$y = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0, \quad n \in \mathbb{N} \cup \{0\}$$

where $a_0, a_1, \ldots, a_n$ are numbers called coefficients.

Population [p. 655] the set of all eligible members of a group which we intend to study

Population parameter [p. 659] a statistical measure that is based on the whole population; the value is constant for a given population

Population proportion, p [p. 658] the proportion of individuals in the entire population possessing a particular attribute

Position [p. 316] For a particle moving in a straight line, the position of the particle relative to a point O on the line is determined by its distance from O and whether it is to the right or left of O. The direction to the right of O is positive.

Power function [p. 26] a function of the form $f(x) = x^r$, where r is a non-zero real number

Probability [p. 516] a numerical value assigned to the likelihood of an event occurring. If the event A is impossible, then $\Pr(A) = 0$; if the event A is certain, then $\Pr(A) = 1$; otherwise $0 < \Pr(A) < 1$.

Probability density function [p. 585] usually denoted $f(x)$; describes the probability distribution of a continuous random variable X such that $\Pr(a < X < b) = \int_a^b f(x)\,dx$

Probability function (discrete) [p. 535] denoted by $p(x)$ or $\Pr(X = x)$, a function that assigns a probability to each value of a discrete random variable X. It can be represented by a rule, a table or a graph, and must give a probability $p(x)$ for every value x that X can take.

Probability table [p. 521] a table used for illustrating a probability problem diagrammatically

Product of functions [p. 21] $(fg)(x) = f(x)\,g(x)$ and $\text{dom}(fg) = \text{dom}\, f \cap \text{dom}\, g$

Product rule [p. 298] If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$
In Leibniz notation:
If $y = uv$, then $\dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

Pythagorean identity [p. 138]
$\cos^2\theta + \sin^2\theta = 1$

Q

$\mathbb{Q}$ [p. 3] the set of all rational numbers

Quadratic, turning point form [p. 77] The turning point form of a quadratic function is $y = a(x - h)^2 + k$, where (h, k) is the turning point.

Quadratic formula [p. 81] The solutions of the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Quadratic function [p. 76] A quadratic has a rule of the form $y = ax^2 + bx + c$, where a, b and c are constants and $a \neq 0$.

Quartic function [p. 107] a polynomial of degree 4. A quartic function f has a rule of the form $f(x) = ax^4 + bx^3 + cx^2 + dx + e$, where $a \neq 0$.

Quotient rule [p. 302] If $h(x) = \dfrac{f(x)}{g(x)}$ then

$$h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

In Leibniz notation:
If $y = \dfrac{u}{v}$, then $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

R

$\mathbb{R}^+$ [p. 3] $\{ x : x > 0 \}$, positive real numbers

$\mathbb{R}^-$ [p. 3] $\{ x : x < 0 \}$, negative real numbers

$\mathbb{R} \setminus \{0\}$ [p. 3] the set of real numbers excluding 0

$\mathbb{R}^2$ [p. 46] $\{ (x, y) : x, y \in \mathbb{R} \}$; i.e. $\mathbb{R}^2$ is the set of all ordered pairs of real numbers

Radian [p. 129] One radian (written 1^c) is the angle subtended at the centre of the unit circle by an arc of length 1 unit:

$$1^c = \frac{180^\circ}{\pi} \quad \text{and} \quad 1^\circ = \frac{\pi^c}{180}$$

Random experiment [p. 516] an experiment, such as the rolling of a die, in which the outcome of a single trial is uncertain but observable

Random sample [p. 655] A sample of size n is called a *simple random sample* if it is selected from the population in such a way that every subset of size n has an equal chance of being chosen as the sample. In particular, every member of the population must have an equal chance of being included in the sample.

Random variable [p. 534] a variable that takes its value from the outcome of a random experiment; e.g. the number of heads observed when a coin is tossed three times

Range [p. 6] the set of all the second coordinates of the ordered pairs in a relation

Rational number [p. 3] a number that can be written as $\dfrac{p}{q}$, for some integers p and q with $q \neq 0$

Rectangular hyperbola [p. 27] The basic rectangular hyperbola has equation $y = \dfrac{1}{x}$.

Recurrence relation [MM1&2] a rule which enables each subsequent term of a sequence to be found from previous terms; e.g. $t_1 = 1$, $t_n = t_{n-1} + 2$

Reflection in the x-axis [p. 52] A reflection in the x-axis is described by the rule $(x, y) \to (x, -y)$. The curve with equation $y = f(x)$ is mapped to the curve with equation $y = -f(x)$.

Reflection in the y-axis [p. 52] A reflection in the y-axis is described by the rule $(x, y) \to (-x, y)$. The curve with equation $y = f(x)$ is mapped to the curve with equation $y = f(-x)$.

Regression [p. 236] the process of fitting a mathematical model to data

Relation [p. 6] a set of ordered pairs; e.g. $\{ (x, y) : y = x^2 \}$

Relative growth rate [pp. 222, 313] The relative growth rate of a function f is $\dfrac{f'(x)}{f(x)}$.
For an exponential function $f(x) = Ae^{kx}$, the constant k is the relative growth rate.

Remainder theorem [p. 97] When a polynomial $P(x)$ is divided by $\beta x + \alpha$, the remainder is $P\left(-\dfrac{\alpha}{\beta}\right)$.

Right-endpoint method [p. 373] gives an estimate for the area under the graph of $y = f(x)$ between $x = a$ and $x = b$:

$$R_n = \frac{b - a}{n}\Big[f(x_1) + f(x_2) + \cdots + f(x_n)\Big]$$

S

Sample [p. 655] a subset of the population which we select in order to make inferences about the whole population

Sample proportion, $\hat{p}$ [p. 659] the proportion of individuals in a particular sample possessing a particular attribute. The sample proportions $\hat{p}$ are the values of a random variable $\hat{P}$.

Sample space, ε [p. 516] the set of all possible outcomes for a random experiment

Sample statistic [p. 659] a statistical measure that is based on a sample from the population; the value varies from sample to sample

Sampling distribution [p. 665] the distribution of a statistic which is calculated from a sample

Scientific notation [MM1&2] A number is in standard form when written as a product of a number between 1 and 10 and an integer power of 10; e.g. 6.626×10^{-34}.

Secant [p. 254] a straight line that passes through two points $(a, f(a))$ and $(b, f(b))$ on the graph of a function $y = f(x)$

Second derivative [p. 441]

- The second derivative of a function f with rule $f(x)$ is denoted by f'' and has rule $f''(x)$.
- The second derivative of y with respect to x is denoted by $\frac{d^2y}{dx^2}$.

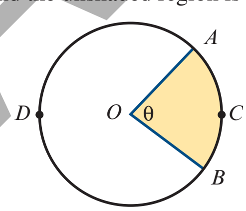
Second derivative test [p. 449]

- If $f'(a) = 0$ and $f''(a) > 0$, then the point $(a, f(a))$ is a local minimum.
- If $f'(a) = 0$ and $f''(a) < 0$, then the point $(a, f(a))$ is a local maximum.
- If $f''(a) = 0$, then further investigation is necessary.

Sector [MM1&2] Two radii and an arc define a region called a sector. In this diagram, the shaded region is a *minor sector* and the unshaded region is a *major sector*.

Area of sector $= \frac{1}{2}r^2\theta$

where $\theta^c = \angle AOB$



Selections [p. 744] counted when order is not important. The number of ways of selecting r objects from a total of n objects is

$${}^nC_r = \frac{n!}{r!\,(n-r)!}$$

An alternative notation for ${}^nC_r$ is $\binom{n}{r}$.

Sequence [MM1&2] a list of numbers, with the order being important; e.g. 1, 1, 2, 3, 5, 8, 13, . . . The numbers of a sequence are called its *terms*, and the nth term is often denoted by t_n.
see also arithmetic sequence, geometric sequence

Series [MM1&2] the sum of the terms in a sequence
see also arithmetic series, geometric series

Set difference [p. 3] The set $A \setminus B$ contains all the elements of A that are not in B. For example, $\mathbb{R} \setminus \{0\}$ is the set of all real numbers excluding 0.

Set notation [p. 2]

- $\in$ means 'is an element of'
- $\notin$ means 'is not an element of'
- $\subseteq$ means 'is a subset of'
- $\cap$ means 'intersection'
- $\cup$ means 'union'
- $\varnothing$ is the empty set, containing no elements

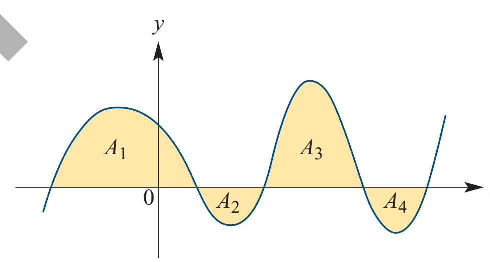
Sets of numbers [p. 3]

- $\mathbb{N}$ is the set of natural numbers
- $\mathbb{Z}$ is the set of integers
- $\mathbb{Q}$ is the set of rational numbers
- $\mathbb{R}$ is the set of real numbers

Signed area [p. 383]

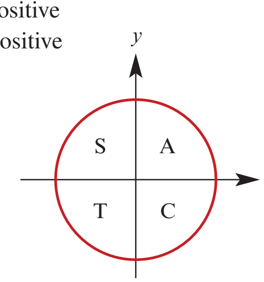
- Regions *above* the x-axis are defined to have *positive* signed area.
- Regions *below* the x-axis are defined to have *negative* signed area.

For example, the signed area of the shaded region in the following graph is $A_1 - A_2 + A_3 - A_4$.



Signs of trigonometric functions [p. 134]

1st quadrant	all are positive
2nd quadrant	sin is positive
3rd quadrant	tan is positive
4th quadrant	cos is positive



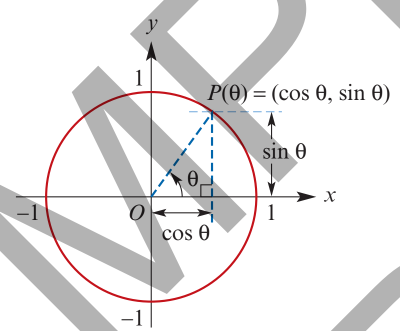
Simple interest [MM1&2] is always calculated on the amount originally invested (the *principal*). If $\$P$ is invested at $R\%$ p.a., then the value of the investment after n years, $\$A_n$, is given by

$$A_n = P + nP\frac{R}{100}$$

Simulation [p. 660] using technology (calculators or computers) to repeat a random process many times; e.g. random sampling

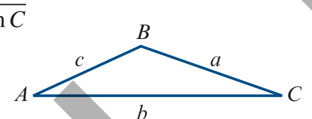
Simultaneous equations [p. 117] equations of two or more lines or curves in the Cartesian plane, the solutions of which are the points of intersection of the lines or curves

Sine function [p. 131] $\sin\theta$ is defined as the y-coordinate of the point P on the unit circle where OP forms an angle of θ radians with the positive direction of the x-axis.



Sine rule [p. 486] For triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The sine rule is used to find unknown quantities in a triangle given one side and two angles, or given two sides and a non-included angle.

Speed [p. 317] the magnitude of velocity

Speed, average [p. 317]

$$\text{average speed} = \frac{\text{total distance travelled}}{\text{total time taken}}$$

Standard deviation of a random variable, σ [pp. 545, 602] a measure of the spread or variability, given by $\mathrm{sd}(X) = \sqrt{\mathrm{Var}(X)}$

Standard form [MM1&2] *see* scientific notation

Standard normal distribution [p. 623] a special case of the normal distribution where $\mu = 0$ and $\sigma = 1$

Stationary point [p. 322] A point $(a, f(a))$ on a curve $y = f(x)$ is a stationary point if $f'(a) = 0$.

Straight line, equation given two points [p. 43] $y - y_1 = m(x - x_1)$, where $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Straight line, gradient–intercept form [p. 43] $y = mx + c$, where m is the gradient and c is the y-axis intercept

Straight lines, parallel [MM1&2] Two non-vertical straight lines are parallel to each other if and only if they have the same gradient.

Straight lines, perpendicular [p. 43] Two straight lines are perpendicular to each other if and only if the product of their gradients is -1 (or if one is horizontal and the other vertical).

Strictly decreasing [pp. 26, 265] A function f is strictly decreasing on an interval if $x_2 > x_1$ implies $f(x_2) < f(x_1)$.

Strictly increasing [pp. 26, 265] A function f is strictly increasing on an interval if $x_2 > x_1$ implies $f(x_2) > f(x_1)$.

Subset [p. 2] A set B is called a subset of set A if every element of B is also an element of A. We write $B \subseteq A$.

Sum of functions [p. 21] $(f + g)(x) = f(x) + g(x)$ and $\mathrm{dom}(f + g) = \mathrm{dom}\, f \cap \mathrm{dom}\, g$

Sum of two cubes [p. 99] $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Sum to infinity [MM1&2] The sum to infinity of an infinite geometric series exists provided $-1 < r < 1$ and is given by

$$S_\infty = \frac{a}{1 - r}$$

where $a = t_1$ and r is the common ratio.

T

Tangent function [p. 131] The tangent function is given by

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Tangent line, equation [p. 305] Let (x_1, y_1) be a point on the curve $y = f(x)$. Then, if f is differentiable at $x = x_1$, the equation of the tangent at (x_1, y_1) is given by $y - y_1 = f'(x_1)(x - x_1)$.

Total change [p. 405] Given the rule for $f'(x)$, the total change in the value of $f(x)$ between $x = a$ and $x = b$ can be found using

$$f(b) - f(a) = \int_a^b f'(x)\, dx$$

Translation [p. 46] A translation of h units in the positive direction of the x-axis and k units in the positive direction of the y-axis is described by the rule $(x, y) \to (x + h, y + k)$, where $h, k > 0$. The curve with equation $y = f(x)$ is mapped to the curve with equation $y - k = f(x - h)$.

Trapezoidal rule [p. 375] gives an estimate for the area under the graph of $y = f(x)$ between $x = a$ and $x = b$:

$$T_n = \frac{b-a}{2n}\left[f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right]$$

Tree diagram [p. 527] a diagram representing the outcomes of a multi-stage experiment

Trigonometric functions [p. 131] the sine, cosine and tangent functions

Trigonometric functions, exact values [p. 133]

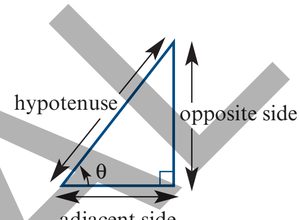
θ^c	θ°	$\sin\theta$	$\cos\theta$	$\tan\theta$
0	0°	0	1	0
$\frac{\pi}{6}$	30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$\frac{\pi}{4}$	45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$\frac{\pi}{3}$	60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$\frac{\pi}{2}$	90°	1	0	undefined

Trigonometric ratios [p. 481]

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$



U

Uniform distribution [pp. 558, 621]

- A discrete random variable X with n values $x_1, x_2, x_3, \ldots, x_n$ has a uniform distribution if each value of X is equally likely, and therefore

  $$\Pr(X = x) = \frac{1}{n}, \text{ for } x = x_1, x_2, x_3, \ldots, x_n$$

- A continuous random variable X has a uniform distribution if its probability density function is given by

  $$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{if } a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

  where a and b are real constants with $a < b$.

Union of sets [pp. 2, 516] The union of two sets A and B, written $A \cup B$, is the set of all elements which are in A or B or both.

V

Variance of a random variable, σ^2 [pp. 545, 602] a measure of the spread or variability, defined by $\text{Var}(X) = \text{E}[(X - \mu)^2]$. An alternative (computational) formula is $\text{Var}(X) = \text{E}(X^2) - [\text{E}(X)]^2$

Velocity [p. 317] the rate of change of a particle's position with respect to time

Velocity, average [p. 317]

$$\text{average velocity} = \frac{\text{change in position}}{\text{change in time}}$$

Velocity, instantaneous [p. 317] $v = \dfrac{dx}{dt}$

Vertical-line test [p. 8] *see* function

Z

$\mathbb{Z}$ [p. 3] the set of all integers

Zero polynomial [p. 90] The number 0 is called the zero polynomial.