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# MATHEMATICAL METHODS

## UNITS 3 & 4

CAMBRIDGE SENIOR MATHEMATICS  
FOR QUEENSLAND

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INCLUDES INTERACTIVE  
TEXTBOOK POWERED BY  
CAMBRIDGE HOTMATHS



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**Online assessment practice in the Interactive Textbook and Online Teaching Suite**

IA1: Practice problem-solving and modelling task

IA2: Practice internal examination on Unit 3

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**Online assessment practice in the Interactive Textbook and Online Teaching Suite**

IA3: Practice internal examination on Unit 4

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**Online Appendix B: Guides to using technology**

These online guides are accessed through the Interactive Textbook or PDF Textbook

- B1 Online guide to the TI-Nspire CX Non-CAS graphics calculator
- B2 Online guide to the TI-84 Plus CE graphics calculator
- B3 Online guide to the Casio fxCG20AU and Casio fxCG50AU graphics calculators
- B4 Online guide to the Desmos graphing calculator

*Note: A printable copy of the **QCAA Formula sheet** is available in the Interactive Textbook*



# About the lead author and consultants

## About the lead author

**Michael Evans** was a consultant to ACARA on the writing of the Australian Curriculum on which the new Queensland syllabus is based. He is a consultant with the Australian Mathematical Sciences Institute, and is coordinating author of the ICE-EM 7–10 series also published by Cambridge.

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**Trevor Redmond** is Head of Mathematics at Somerville House, South Brisbane

**Ray Minns** is Head of Mathematics at Northpine Christian College, Dakabin, QLD

# Introduction and overview

*Cambridge Senior Mathematics for Queensland Mathematical Methods Units 3 & 4* provides complete and close coverage of the QCAA syllabus to be implemented in Year 12 from 2020. Its four components — the print book, downloadable PDF textbook, online Interactive Textbook (ITB) and Online Teaching Resource (OTS), both powered by the HOTmaths platform — contain a huge range of resources, including worked solutions available to schools in a single package at one convenient price (the OTS is included with class adoptions, conditions apply). There are no extra subscriptions or per-student charges to pay.

**Preliminary topics (review of Units 1&2):** The first four chapters can be considered as a review of Units 1&2: *Chapter 1 Functions and relations*, *Chapter 2 Coordinate geometry and transformations*, *Chapter 3 Polynomial functions* and *Chapter 4 Trigonometric functions*. The topics covered in these chapters are important for Units 3&4 and of course may be examined at year 12. You may choose to complete these chapters prior to the beginning of Year 12.

In addition, two ‘refresher’ chapters are provided: *Chapter 7 Refresher on differentiation* and *Chapter 14 Refresher on probability and discrete random variables*. It is recommended that these be done just before the chapters for which they are preparation.

To help decide whether any students can be exempted from doing the preliminary topics and refresher chapters, the multiple-choice question sections from their chapter reviews are set up in the Online Teaching Suite to provide the option of being used as diagnostic tests for this purpose.

**Degree of difficulty** classification of questions: in the exercises, questions are classified as simple familiar **SF**, complex familiar **CF**, and complex unfamiliar **CU** questions. The revision chapters described below also contain model questions for each of these categories, and tests are also provided in the teacher resources, made up of such categorised model questions.

**Three revision chapters of material covered in Units 3 and 4:** These chapters contain sections on *Technology-free questions*, *Multiple choice questions*, *Extended-response questions*, and *Degree of difficulty classification of questions*. The first revision chapter occurs at the end of Unit 3, the second at the end of Unit 4 and there is a final revision chapter that will help with revision for the external examination

**Calculator guidance:** Throughout the book there is guidance for the use of the TI-Nspire CX non-CAS and the Casio fxCG20AU and fxCG50AU graphics calculators for the solution of problems. Guidance on the TI-84Plus CE is included in the interactive textbook, accessed via icons next to the TI-Nspire boxes. There are also online guides for the general use of each of these calculators.

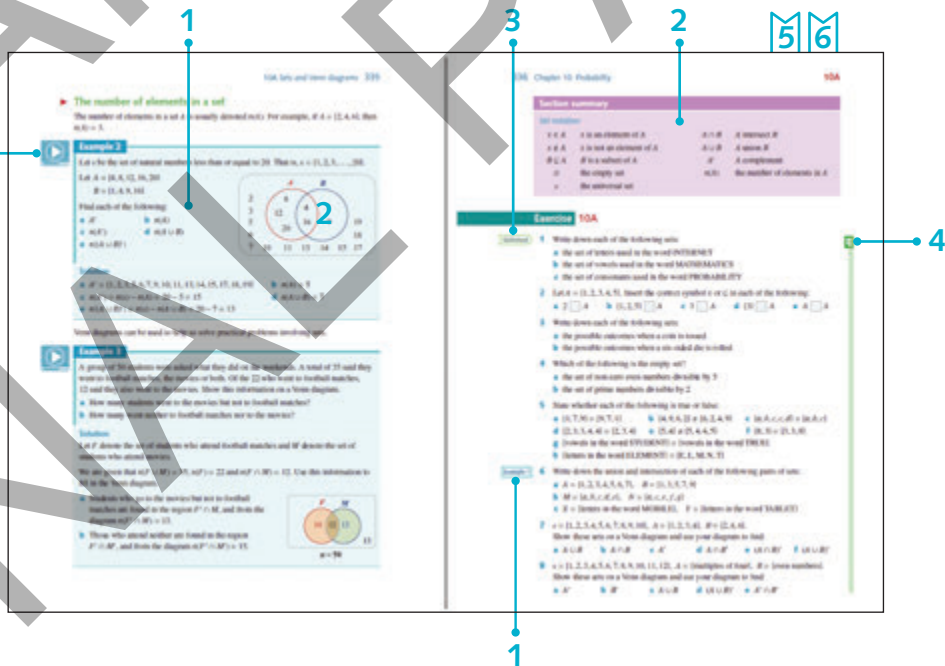
The online graphing calculator from Desmos.com is also embedded in the interactive textbook, as blank screens that students and teachers can use for their own calculations, or as widgets which have been set up for a variety of activities. The new Desmos geometry tool is also embedded in the Interactive Textbook, and activities and widgets using the tool will be added as they are developed.

**Assessment practice:** two sets of problem-solving and modelling tasks and internal and external examinations are provided, one in the Interactive Textbook which students can access, and a different set in the Online Teaching Suite for teacher-only access.

► **Overview of the print book (shown below)**

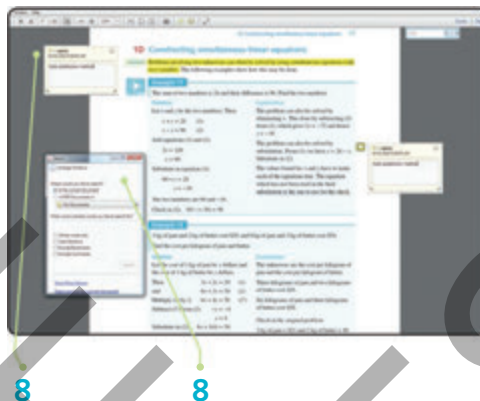
- 1 Graded step-by-step worked examples with precise explanations (and video versions) encourage independent learning, and are linked to exercises.
- 2 Section summaries provide important concepts in boxes for easy reference.
- 3 Additional linked resources in the Interactive Textbook are indicated by icons, such as skillsheets and video versions of examples.
- 4 Degree of difficulty categories are indicated in exercises (similar familiar, complex familiar and complex unfamiliar).
- 5 Chapter reviews contain a chapter summary and technology-free, multiple-choice, and extended-response questions. The latter are classified by degree of difficulty.
- 6 The glossary includes page numbers of the main explanation of each term.

*Numbers refer to descriptions above. Content shown from Units 1 & 2.*



### ► Overview of the downloadable PDF textbook

- 7 The convenience of a downloadable PDF textbook has been retained for times when users cannot go online.
- 8 PDF annotation and search features are enabled.



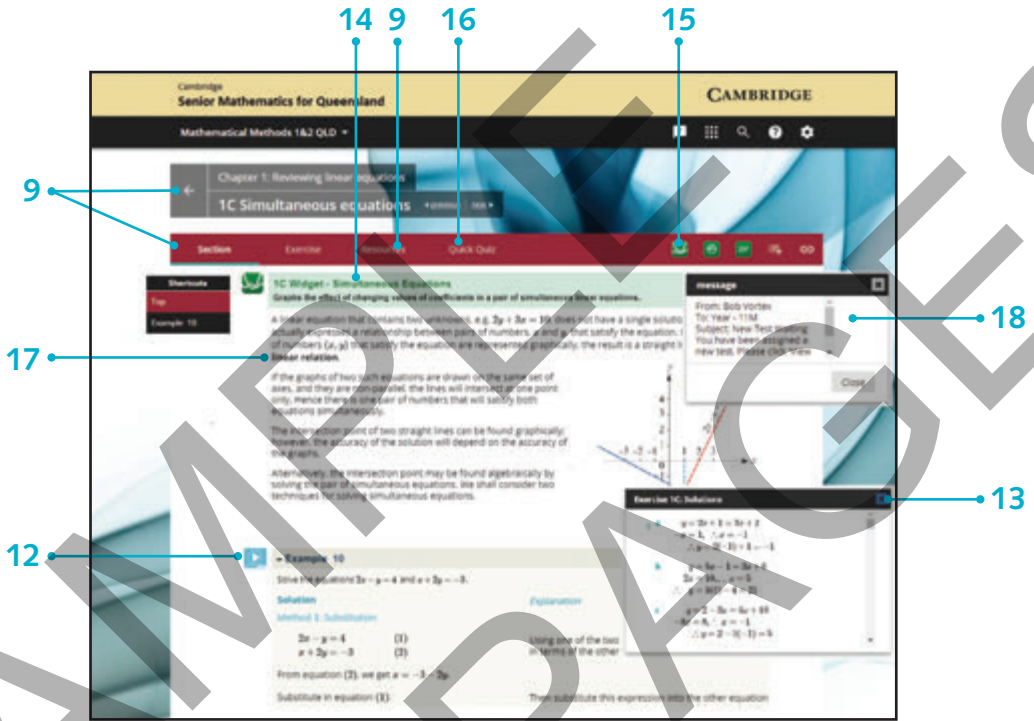
### ► Overview of the Interactive Textbook (shown on the page opposite)

The **Interactive Textbook (ITB)** is an online HTML version of the print textbook powered by the HOTmaths platform, included with the print book or available as a separate purchase.

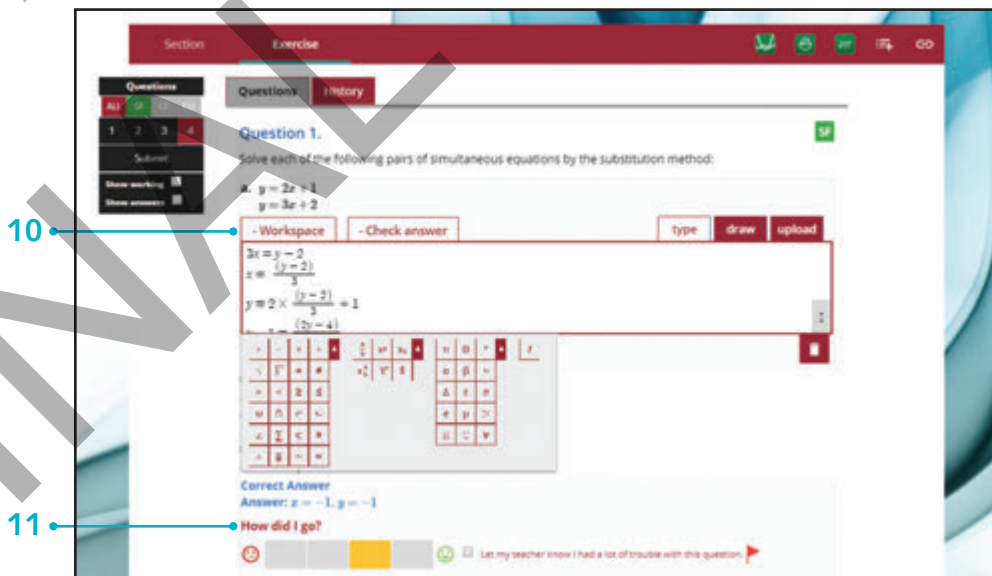
- 9 The material is formatted for on screen use with a convenient and easy-to-use navigation system and links to all resources.
- 10 The new **Workspaces** enable students to enter working and answers online and to save them. Input is by typing, with the help of a symbol palette, handwriting and drawing on tablets, or by uploading images of writing or drawing.
- 11 The new **self-assessment tools** enable students to check answers, mark their own work, and rate their confidence level in their work. This helps develop responsibility for learning and communicates progress and performance to the teacher. Student accounts can be linked to the learning management system used by the teacher in the Online Teaching Suite, so that teachers can review student self-assessment and provide feedback or adjust marks.
- 12 Examples have **video versions** to encourage independent learning.
- 13 **Worked solutions** are included and can be enabled or disabled in the student ITB accounts by the teacher.
- 14 Interactive **Desmos widgets** and activities based on embedded graphics calculator and geometry tool windows demonstrate key concepts and enable students to visualise the mathematics.
- 15 The **Desmos graphics calculator, scientific calculator, and geometry tool** are also embedded for students to use for their own calculations and exploration.
- 16 **Quick quizzes** containing automarked multiple choice questions enable students to check their understanding.
- 17 **Definitions** pop up for key terms in the text, and are also provided in a **dictionary**.
- 18 Messages from the teacher assign tasks and tests.
- 19 **Assessment practice** items for student access are provided in downloadable PDF files.
- 20 Online guides to technology are provided for three calculator models and Desmos.

INTERACTIVE TEXTBOOK POWERED BY THE HOTmaths PLATFORM 

A selection of features is shown. Numbers refer to the descriptions on the opposite page. HOTmaths platform features are updated regularly. Content shown from Units 1 & 2.



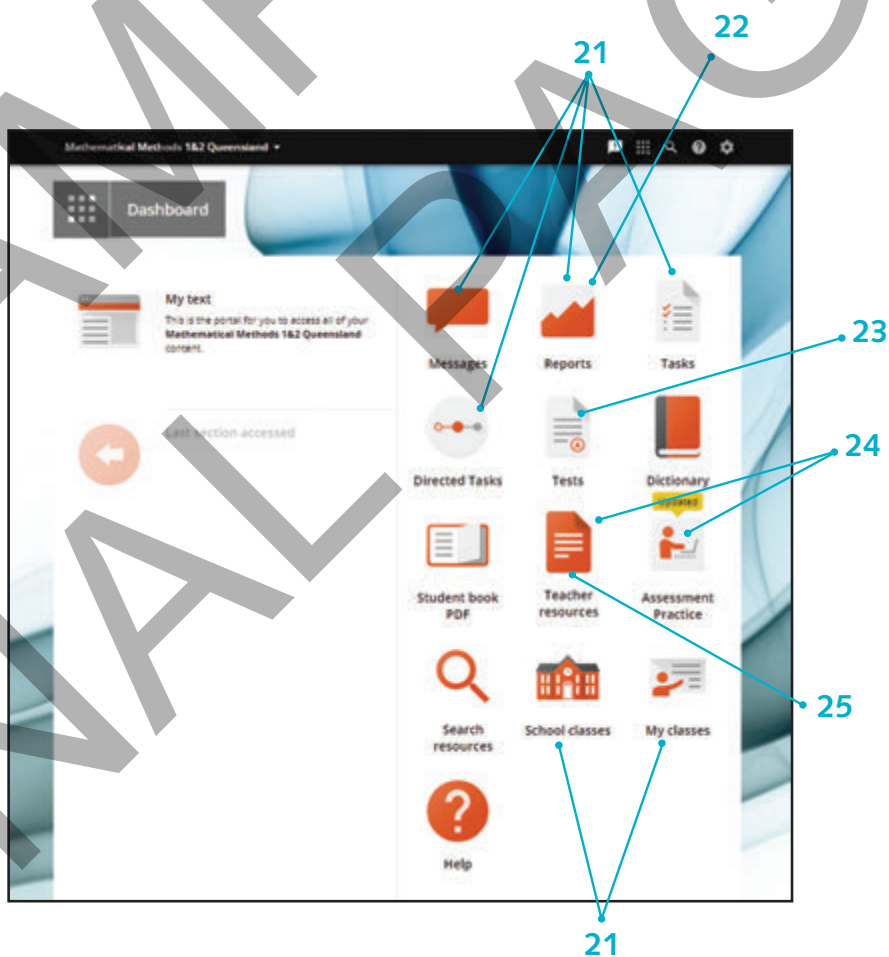
WORKSPACES AND SELF-ASSESSMENT



► **Overview of the Online Teaching Suite powered by the HOTmaths platform (shown below)**

The Online Teaching Suite is automatically enabled with a teacher account and is integrated with the teacher’s copy of the Interactive Textbook. All the teacher resources are in one place for easy access. The features include:

- 21** The HOTmaths learning management system with class and student analytics and reports, and communication tools.
- 22** Teacher’s view of a student’s working and self-assessment which enables them to modify the student’s self-assessed marks, and respond where students flag that they had difficulty.
- 23** A HOTmaths-style test generator.
- 24** Chapter test worksheets and teachers’ set of assessment practice items (these are listed in the table of contents of this textbook).
- 25** Editable curriculum grids and teaching programs.



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